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Calculus 1210-400

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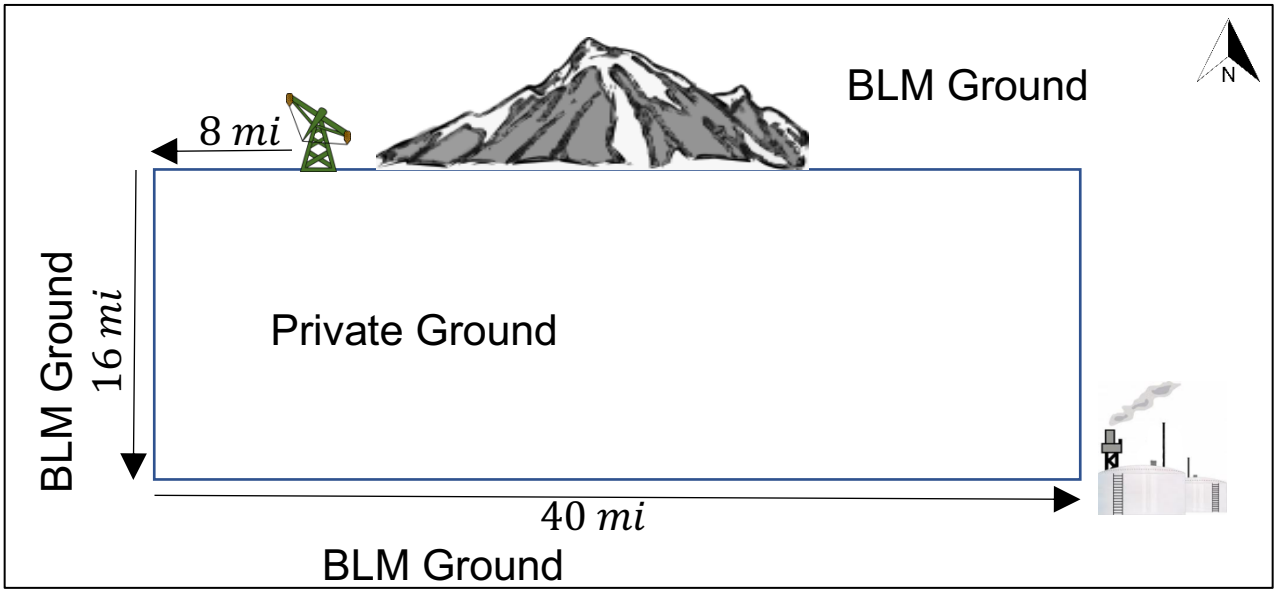
Pipeline Project

We need to run a pipeline from the drill site to the refinery. I have prepared several options illustrating direction of construction and cost of the project with each option.

Option A: Run the pipeline strictly on BLM Ground

Route 1—Run west, south, then east to the refinery.



Fees, materials, and labor (Normal Cost): \$480,000 *per mile*

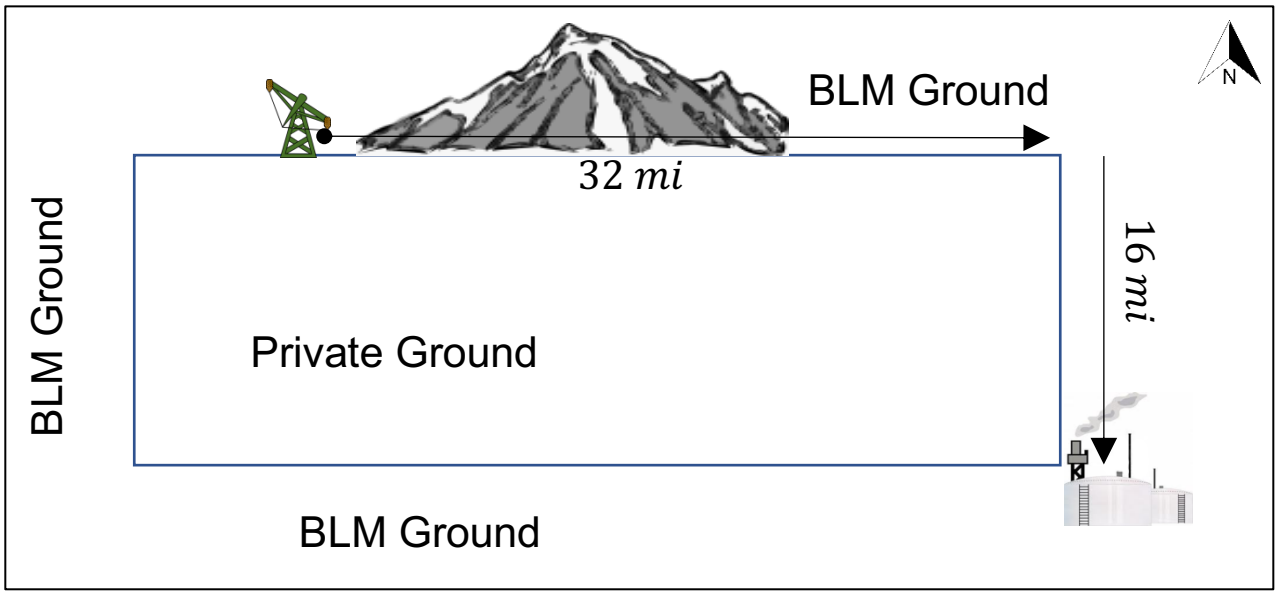
Miles: $8\ mi\ West\ +\ 16\ mi\ South\ +\ 40\ mi\ East\ =\ 64\ mi$

$$Total\ Cost = Normal\ Cost \times miles$$

$$Total\ Cost = \$480{,}000 \times 64\ mi = \$30{,}720{,}000$$

With this option, we run a simple partial perimeter around Private Ground to the Refinery in the direction opposite the mountain, at a cost of \$30,720,000.

Route 2—Run East, go through the mountain, then South to the refinery.



Normal cost: $\$480{,}000\ per\ mi$

Miles: $32\ mi\ East + 16\ mi\ South = 48\ mi$

One-time (OT) cost: $\$4{,}500{,}000$

Environmental Impact (EI) Study: $\$600{,}000\ study, 8\ mo\ delay$

Delay: $\$100{,}000\ per\ mo$

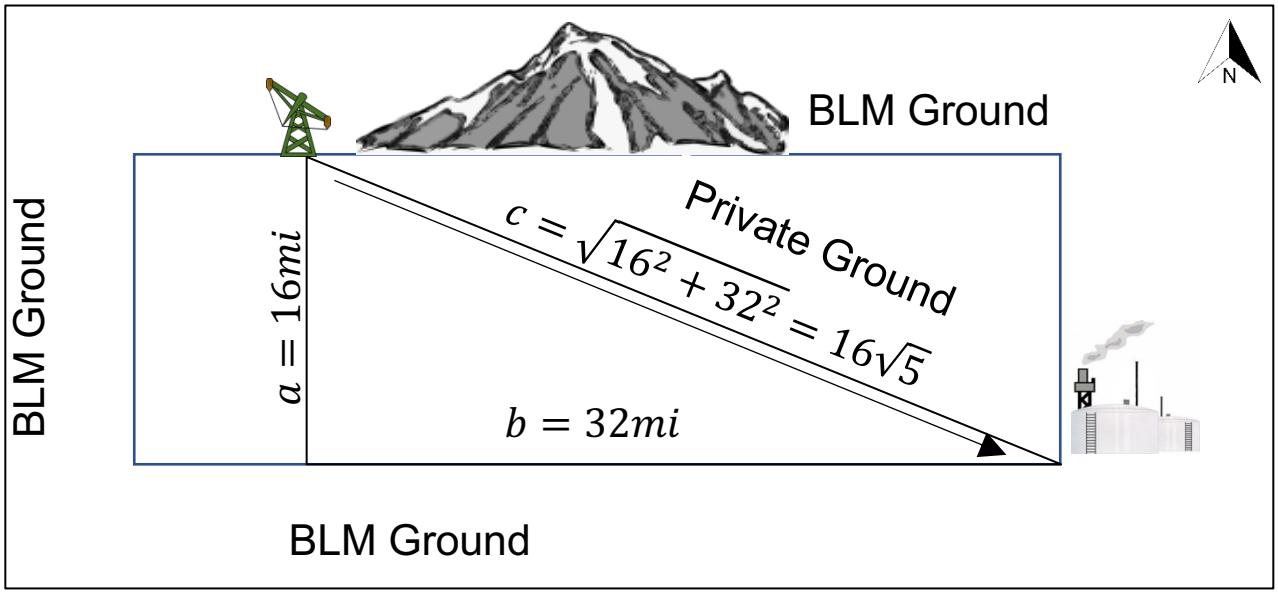
$$Total\ Cost = (Normal\ Cost \times miles) + OT\ Cost + EI\ Study + (Delay \times months)$$

$$Total\ Cost = (\$480{,}000 \times 48) + \$4{,}500{,}000 + \$600{,}000 + (\$100{,}000 \times 8) = \$28{,}940{,}000$$

This option takes us through another partial perimeter, this time going in the opposite direction, and through the mountain. Here we account for the cost differential incurred by having to drill (baby, drill) into the mountain, fund an Environmental Impact Study required by BLM, and the cost of delaying the project for eight months, which is the minimum estimated time that the Environmental Impact Study requires. Going this route would cost $28,940,000.

Option B: Run the pipeline through private ground.

Route 3: Run the shortest distance across the private ground to the refinery.



Normal cost: $\$480{,}000\ per\ mi$

Right-of-way (ROW) fee: $\$360{,}000\ per\ mi$

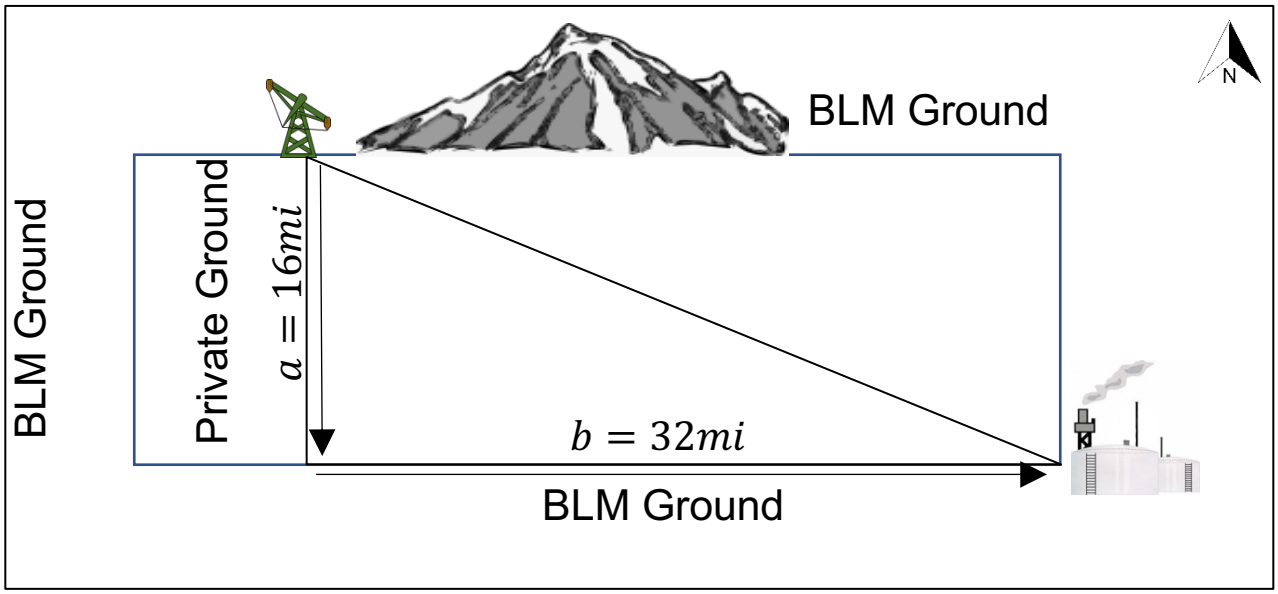
Miles: $16\sqrt{5}\ mi$

$$Total\ Cost = (Normal\ cost + ROW\ fee) \times Miles$$

$$Total\ Cost = (\$480{,}000 + \$360{,}000) \times 16\sqrt{5} = 13{,}440{,}000\sqrt{5} \approx \$30{,}052{,}753.62$$

This option has us taking the shortest possible route to the Refinery, and completely through Private Ground. Aside from Normal cost, here we must account for the added cost of Right-of-way fees. This route would cost $30,052,754.

Route 4: Run straight South across Private Ground, then straight East to the refinery.



Normal cost: $\$480{,}000\ per\ mi$

ROW fee: $\$360{,}000\ per\ mi$

Miles $_{\text{ROW}}$: $16\ mi$

Miles $_{\text{BLM}}$: $32\ mi$

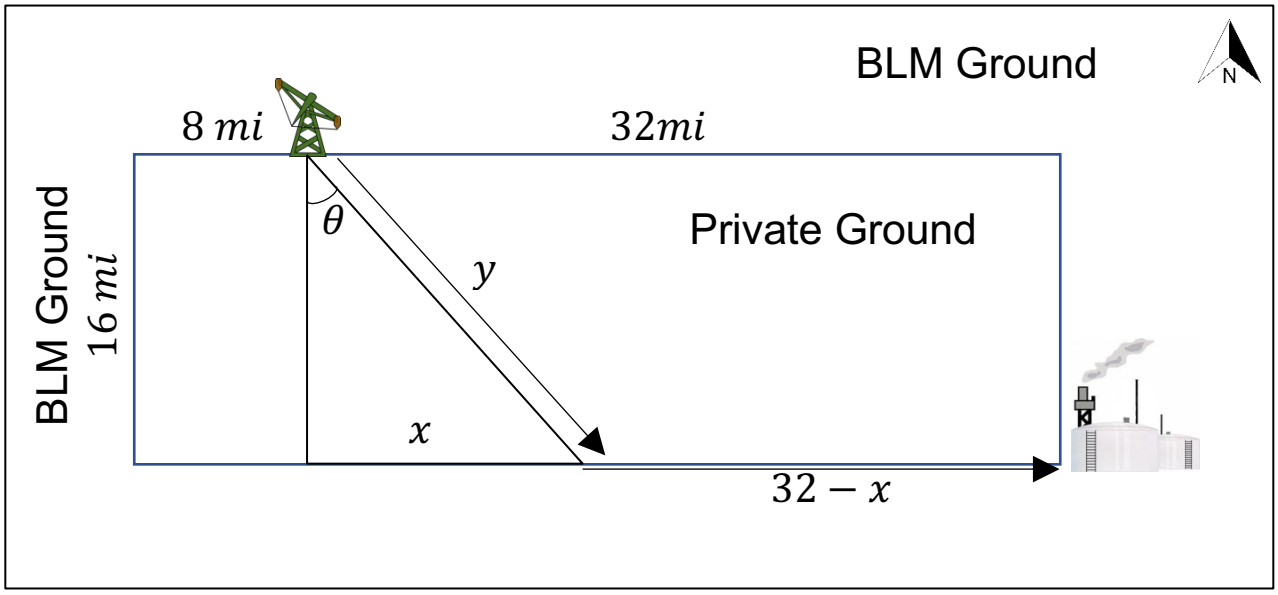
$$Total\ Cost = [(Normal\ cost + ROW\ fee) \times Miles_{ROW}] + [Normal\ cost \times Miles_{BLM}]$$

$$Total\ Cost = [(\$480{,}000 + \$360{,}000) \times 16\ mi] + [\$480{,}000 \times 32\ mi] = \$28{,}800{,}000$$

This option has us taking the shortest possible route through Private Grounds to hopefully cut down on Right-of-way fees, and going the rest of the way to the Refinery on BLM land. This turned out to be our second best option for cutting costs at $28,800,000.

Option C: Use Calculus to find a function that would minimize cost.

Route 5: Run Southeast from the well at some angle θ, intersect BLM Ground to the South, and then run East to the Refinery.



Formulas for Angle in radians and Distance in miles:

$$tan\theta = \frac{x}{16}$$

$$D_{ROW} = y = \sqrt{x^2 + 16^2} \ \ (arrived\ at\ via\ the\ pythagorean\ theorem)$$

$$D_{BLM} = D = 32 - x$$

Cost Function $C(x)$ in dollars:

$$C(x) = 480{,}000(32 - x) + 840{,}000y$$

Substitute to get one variable:

$$C(x) = 480{,}000(32 - x) + 840{,}000\sqrt{x^2 + 16^2}$$

$$C(x) = 15{,}360{,}000 - 480{,}000x + 840{,}000\sqrt{x^2 + 16^2}$$

The first derivative of $C(x) = C'(x)$, used to find critical points, if any:

$$C'(x) = (-480{,}000) + [840{,}000 \times \frac{1}{2} \times (x^2 + 16^2)^{-\frac{1}{2}} \times 2x]$$

$$C'(x) = \frac{840{,}000x}{\sqrt{x^2 + 16^2}} - 480{,}000$$

Find where $C'(x) = 0$:

$$C'(x) = \frac{840{,}000x}{\sqrt{x^2 + 16^2}} - 480{,}000 = 0$$

$$\frac{840{,}000x}{\sqrt{x^2 + 16^2}} = 480{,}000$$

$$\frac{840{,}000x}{480{,}000} = \sqrt{x^2 + 16^2}$$

$$\frac{840{,}000x}{480{,}000} = \sqrt{x^2 + 16^2}$$

$$\frac{7x}{4} = \sqrt{x^2 + 16^2}$$

$$\frac{49x^2}{16} = x^2 + 16^2$$

$$\frac{49x^2}{16} = x^2 + 16^2$$

$$49x^2 = 16x^2 + 16^3$$

$$33x^2 = 16^3 = 4096$$

$$x^2 = \frac{4096}{33}$$

$$x = \sqrt{\frac{4096}{33}} = \frac{64\sqrt{33}}{33}$$

Find the Angle:

$$tan\theta = \frac{\frac{64\sqrt{33}}{33}}{16} = \frac{4\sqrt{33}}{33}$$

$$\theta = \tan^{-1}\frac{4\sqrt{33}}{33} \approx 0.6082455789\, radians$$

$$\theta = 0.6082455789\, rads \times \frac{180}{\pi} = 34.85°$$

Find Distance (y) across Private Grounds:

$$y = \sqrt{(\frac{64\sqrt{33}}{33})^2 + 16^2} \approx 19.5\, mi$$

Find Distance (D) across BLM Grounds:

$$D = 32 - x$$

$$D = 32 - \frac{64\sqrt{33}}{33}$$

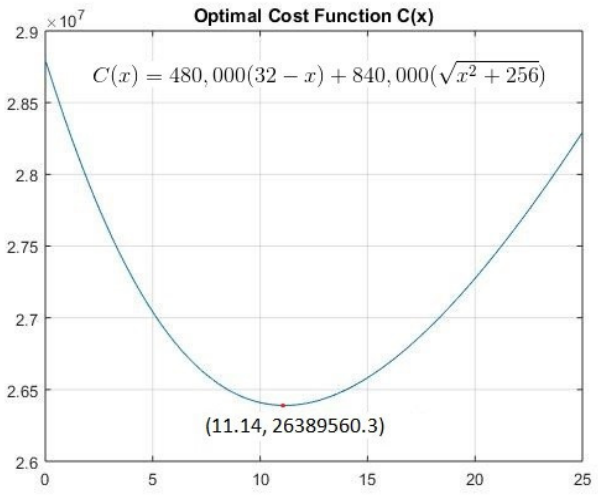
$$D \approx 20.859\, mi$$

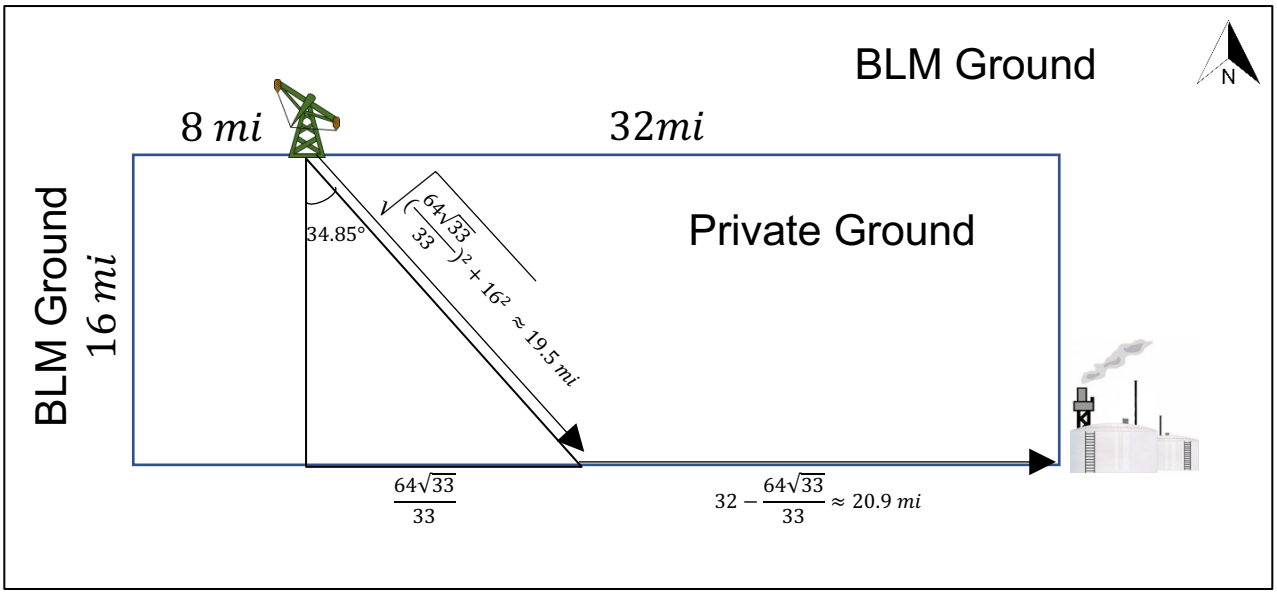
Find minimum cost (C):

$$C = C(x) = 480{,}000(32 - x) + 840{,}000y$$

$$C = 480{,}000\left(32 - \frac{64\sqrt{33}}{33}\right) + 840{,}000\left(\sqrt{(\frac{64\sqrt{33}}{33})^2 + 16^2}\right)$$

$$C = \$26{,}389{,}560.28$$





After using the first derivative of my cost function C(x) to find a critical point, I found that Cost is minimized if we run the pipeline about 34.85 degrees southwest from the well through Private land toward BLM land, then run the pipeline the rest of the way through BLM land. We will build approximately19.5 miles of pipeline on Private Ground, and approximately 20.9 miles of pipeline on BLM land. The minimum possible cost of building this pipeline is $26,389,560.28, which is 2,410,439.72 less than our next best option (Route 4).

Reflection

In my calculus class I have learned about limits, derivatives as limits, shortcuts for finding derivatives, applications of derivatives, antiderivatives as limits, how to use antiderivatives to find areas between curves and the x-axis, and some applications of antiderivatives. On the first day of class, Dr. Mitchell told us that Calculus is rocket science. I liked that. I plan to go into a Geological Engineering. I am not too far into my major, so I cannot be specific about how I think this will apply, but at the very least I think this and other math and science courses in general help me to develop my critical thinking, problem solving, and logic skills. It's about mental agility, I suppose, and all the many problems I have had to work out for this and past mathematics courses have all helped me grow in these areas by leaps and bounds. I expect without a doubt I will need to have those skills developed as a future expert in the field of Geological Engineering.

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